13. Long problem 1. Eagles on the Caraiman Cross !

The tallest cross built on a mountain peak is located on a plateau situated on the top of the peak called Caraiman in Romania at altitude $H = 2300\,m$ from the sea level. Its height, including the base-support is $h = 39,3\,m$. The horizontal arms of the cross are oriented on the N-S direction. The latitude at which the Cross is located is $\varphi = 45^\circ$.

A. On the evening of 21[st] of March 2014, the vernal equinox day, two eagles stop from their flight, first near the monument, and the second, on the top of the Cross as seen in figure 1. The two eagles are in the same

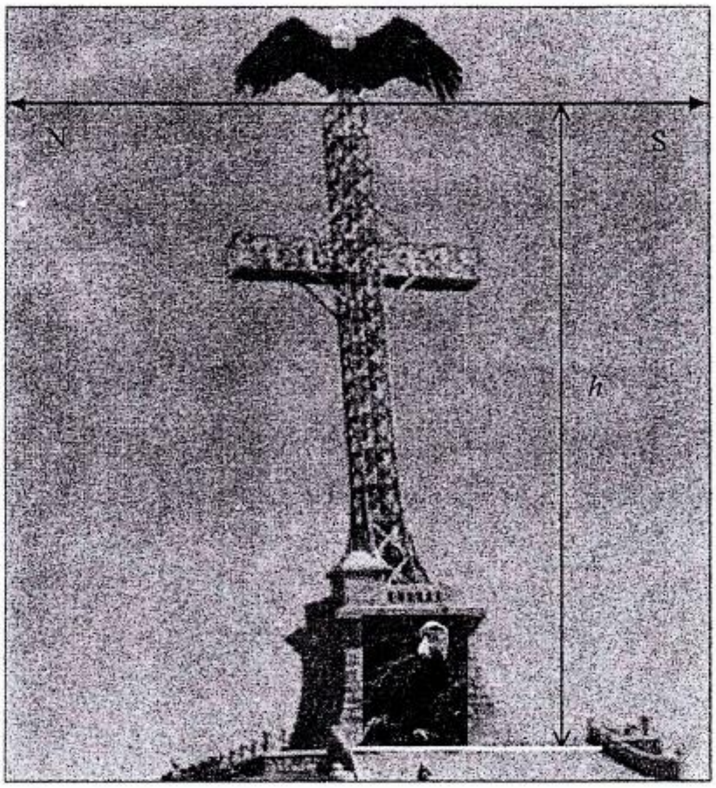

Figure 1

vertical direction. The sky was very clear, so the eagles could see the horizon and observe the Sun set. Each eagle began to fly right at the moment it observed that the Sun disappeared completely.

At the same time, an astronomer is located at the sea-level, at the base of the Bucegi Mountains. Assume that he is in the same vertical direction as the two eagles.

Assuming the atmospheric refraction to be negligible, solve the following questions:

1) *Calculate* the duration of the sunset, measured by the astronomer.

2) *Calculate* the durations of sunsets measured by each of the two eagles and indicate which of the eagles leaves the Cross first. What is the time interval between the moments of the flights of the two eagles.

The following information is necessary:

The duration of the sunset measurement starts when the solar disc is tangent to the horizon line and stops when the solar disc completely disappears.

The Earth's rotational period is $T_E = 24\,\mathrm{h}$, the radius of the Sun $R_S = 6{,}96 \cdot 10^5\,\mathrm{km}$, Earth – Sun distance $d_{ES} = 14.96 \cdot 10^7\,\mathrm{km}$, the latitude of the Heroes Cross is $\varphi = 45^\circ$. $R_E = 6370\,\mathrm{km}$

B) At a certain moment the next day, 22nd March 2014, the two eagles come back to the Heroes Cross. One of the eagles lands on the top of the vertical pillar of the Cross and the other one land on the horizontal plateau, just at the tip of the shadow of the vertical pillar of the Cross, at that moment of the day when the shadow length is minimum.

1) Calculate the distance between the two eagles and the second eagle's distance from the cross.

2) Calculate the length of the horizontal arms of the Cross l_b, if the shadow on the plateau of one of the arm of the cross at this moment has the length $u_b = 7\,\mathrm{m}$

C) At midnight, the astronomer visits the cross and, from its top, he identifies a bright star at the limit of the circumpolarity. He named this star „Eagles Star". Knowing that due to the atmospheric refraction the horizon lowering is $\xi = 34'$, calculate:

1) The "Eagles star" declination;

2) The "Eagles star" maximum height above the horizon.

14. Long problem 2. From Romania .... to Antipod! ...with a ballistic messenger

The 8th IOAA organizers plan to send to the **antipode** (the point on the Earth's surface diametrically opposite to the launch position) the official flag using a ballistic projectile. The projectile will be launched from Romania, and the rotation of the Earth will be neglected.

a) Calculate the coordinates of the target-point if the launch-point coordinates are: $\varphi_{\text{Romania}} = 44^\circ\ North;\ \lambda_{\text{Romania}} = 30^\circ$ Est.

b) Determine the magnitude of the velocity and the launch angle, with respect to the horizon at launch site, in order that the projectile should hit the target.

c) Calculate the velocity of the projectile when it hits the target.

d) Calculate the minimum velocity of the projectile on its trajectory.

e) Calculate the flying –time of the projectile, from the launch to the impact. You may use the value of the gravitational acceleration at Earth surface as $g_0 = 9.81\,\mathrm{ms}^{-2}$; the Earth radius $R = 6370\,\mathrm{km}$.

f) Will it be possible that the projectile will be seen by the naked eye when it is at the maximum distance from the Earth. You will use the following values: The Moon albedo $\alpha_M = 0.12$; The Moon radius $R_M = 1738\,Km$; the Earth –Moon distance $r_{EM} = 385000$ km; the apparent magnitude of the full moon $m_M = -12.7^m$. You assume that the projectile is perfectly metallic sphere with radius $r_{projectile} = 400\,x\,10^{-3}\,m$ and with perfectly reflective surface.